A CANCELLATION-FREE FORMULA FOR THE SCHUR ELEMENTS OF THE ARIKI-KOIKE ALGEBRA

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1. Introduction

Schur elements play a powerful role in the representation theory of symmetric algebras. In the case of the Ariki-Koike algebra, Schur elements are Laurent polynomials whose factors determine when Specht modules are projective irreducible and whether the algebra is semisimple.

Formulas for the Schur elements of the Ariki-Koike algebra have been independently obtained first by Geck, Iancu and Malle [6], and later by Mathas [10]. The aim of this note is to give a cancellation-free formula for these polynomials (Theorem 5.1), so that their factors can be easily read and programmed.

2. Partitions: definitions and notation

A *partition* $\lambda = (\lambda_1, \lambda_2, \lambda_3, \ldots)$ is a decreasing sequence of non-negative integers. We define the *length of* λ to be the smallest integer $\ell(\lambda)$ such that $\lambda_i = 0$ for all $i > \ell(\lambda)$. We write $|\lambda| := \sum_{i\geq 1} \lambda_i$ and we say that λ is a *partition of* m, for some $m \in \mathbb{N}$, if $m = |\lambda|$. We set $n(\lambda) := \sum_{i\geq 1}(i-1)\lambda_i$.

We define the set of nodes $[\lambda]$ of λ to be the set

$$[\lambda] := \{(i,j) \mid i \geq 1,\ 1 \leq j \leq \lambda_i\}.$$

A node $x = (i,j)$ is called *removable* if $[\lambda] \setminus \{(i,j)\}$ is still the set of nodes of a partition. Note that if (i,j) is removable, then $j = \lambda_i$.

The *conjugate partition* of λ is the partition λ' defined by

$$\lambda'_k := \#\{i \,|\, i \geq 1 \text{ such that } \lambda_i \geq k\}.$$

Obviously, $\lambda'_1 = \ell(\lambda)$. The set of nodes of λ' satisfies

$$(i,j) \in [\lambda'] \Leftrightarrow (j,i) \in [\lambda].$$

Note that if (i, λ_i) is a removable node of λ, then $\lambda'_{\lambda_i} = i$. Moreover, we have

$$n(\lambda) = \sum_{i\geq 1}(i-1)\lambda_i = \frac{1}{2}\sum_{i\geq 1}(\lambda'_i - 1)\lambda'_i.$$

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Now, if $x = (i, j) \in [\lambda]$, we define the *content* of x to be the difference

$$\mathrm{cont}(x) = j - i.$$

The following lemma, whose proof is an easy combinatorial exercise (with the use of Young diagrams), relates the contents of the nodes of (the "right rim" of) λ with the contents of the nodes of (the "lower rim" of) λ'.

Lemma 2.1. *Let $\lambda = (\lambda_1, \lambda_2, \ldots)$ be a partition and let k be an integer such that $1 \le k \le \lambda_1$. Let q and y be two indeterminates. Then we have*

$$\frac{1}{(q^{\lambda_1} y - 1)} \cdot \left(\prod_{i=1}^{\lambda'_k} \frac{q^{\lambda_i - i + 1} y - 1}{q^{\lambda_i - i} y - 1} \right) = \frac{1}{(q^{-\lambda'_k + k - 1} y - 1)} \cdot \left(\prod_{j=k}^{\lambda_1} \frac{q^{-\lambda'_j + j - 1} y - 1}{q^{-\lambda'_j + j} y - 1} \right).$$

Finally, if $x = (i, j) \in [\lambda]$ and μ is another partition, we define the *generalized hook length of x with respect to μ* to be the integer:

$$h^{\mu}_{i,j} := \lambda_i - i + \mu'_j - j + 1.$$

For $\mu = \lambda$, the above formula becomes the classical hook length formula (giving us the length of the hook of λ that x belongs to).

3. The Ariki-Koike algebra

Let d and r be positive integers and let R be a commutative domain with 1. Fix elements $q, Q_0, \ldots, Q_{d-1}$ of R, and assume that q is invertible in R. Set $\mathbf{q} := (q; Q_0, \ldots, Q_{d-1})$. The *Ariki-Koike algebra* $\mathcal{H}_{d,r}$ is the unital associative R-algebra with generators $T_0, T_1, \ldots, T_{r-1}$ and relations:

$$\begin{gathered}
(T_0 - Q_0)(T_0 - Q_1) \cdots (T_0 - Q_{d-1}) = 0, \\
(T_i - q)(T_i + 1) = 0 \quad \text{for } 1 \le i \le r - 1, \\
T_0 T_1 T_0 T_1 = T_1 T_0 T_1 T_0, \\
T_i T_{i+1} T_i = T_{i+1} T_i T_{i+1} \quad \text{for } 1 \le i \le r - 2, \\
T_i T_j = T_j T_i \quad \text{for } 0 \le i < j \le r - 1 \text{ with } j - i > 1.
\end{gathered}$$

The Ariki-Koike algebra $\mathcal{H}_{d,r}$ is a deformation of the group algebra of the complex reflection group $G(d, 1, r) = (\mathbb{Z}/d\mathbb{Z}) \wr \mathfrak{S}_r$. Ariki and Koike [2] have proved that $\mathcal{H}_{d,r}$ is a free R-module of rank $d^r r! = |G(d, 1, r)|$. Moreover, Ariki [1] has shown that, when R is a field, $\mathcal{H}_{d,r}$ is (split) semisimple if and only if

$$P(\mathbf{q}) = \prod_{i=1}^{r} (1 + q + \cdots + q^{i-1}) \prod_{0 \le s < t \le d-1} \prod_{-r < k < r} (q^k Q_s - Q_t)$$

is a non-zero element of R.

A *d-partition* of r is an ordered d-tuple $\lambda = (\lambda^{(0)}, \lambda^{(1)}, \ldots, \lambda^{(d-1)})$ of partitions $\lambda^{(s)}$ such that $\sum_{s=0}^{d-1} |\lambda^{(s)}| = r$. Let us denote by $\mathcal{P}(d, r)$ the set of d-partitions of r. In the semisimple case,

Ariki and Koike [2] constructed an irreducible $\mathcal{H}_{d,r}$-module S^λ, called a *Specht module*, for each d-partition λ of r. Further, they showed that $\{S^\lambda \mid \lambda \in \mathcal{P}(d,r)\}$ is a complete set of pairwise non-isomorphic irreducible $\mathcal{H}_{d,r}$-modules. We denote by χ^λ the character of the Specht module S^λ.

Now, there exists a linear form $\tau : \mathcal{H}_{d,r} \to R$ which was introduced by Bremke and Malle in [3], and was proved to be symmetrizing by Malle and Mathas in [8] whenever all Q_i's are invertible in R. An explicit description of this form can be found in any of these two articles. Following Geck's results on symmetrizing forms [5], we obtain the following definition for the Schur elements associated to the irreducible representations of $\mathcal{H}_{d,r}$.

Definition 3.1. *Suppose that R is a field and that $P(\mathbf{q}) \neq 0$. The* Schur elements *of $\mathcal{H}_{d,r}$ are the elements $s_\lambda(\mathbf{q})$ of R such that*

$$\tau = \sum_{\lambda \in \mathcal{P}(d,r)} \frac{1}{s_\lambda(\mathbf{q})} \chi^\lambda.$$

Schur elements play a powerful role in the representation theory of $\mathcal{H}_{d,r}$, as illustrated by the following result (cf. [7, Theorem 7.4.7], [9, Lemme 2.6]).

Theorem 3.2. *Suppose that R is a field. If $s_\lambda(\mathbf{q}) \neq 0$, then the Specht module S^λ is irreducible. Moreover, the algebra $\mathcal{H}_{d,r}$ is semisimple if and only if $s_\lambda(\mathbf{q}) \neq 0$ for all $\lambda \in \mathcal{P}(d,r)$.*

4. Formulas for the Schur elements of the Ariki-Koike algebra

The Schur elements of the Ariki-Koike algebra $\mathcal{H}_{d,r}$ have been independently calculated first by Geck, Iancu and Malle [6], and later by Mathas [10]. From now on, for all $m \in \mathbb{N}$, let $[m]_q := (q^m - 1)/(q-1) = q^{m-1} + q^{m-2} + \cdots + q + 1$. The formula given by Mathas does not demand extra notation and is the following:

Theorem 4.1. *Let $\lambda = (\lambda^{(0)}, \lambda^{(1)}, \ldots, \lambda^{(d-1)})$ be a d-partition of r. Then*

$$s_\lambda(\mathbf{q}) = (-1)^{r(d-1)} (Q_0 Q_1 \cdots Q_{d-1})^{-r} q^{-\alpha(\lambda')} \prod_{s=0}^{d-1} \prod_{(i,j) \in [\lambda^{(s)}]} Q_s [h_{i,j}^{\lambda^{(s)}}]_q \cdot \prod_{0 \leq s < t \leq d-1} X_{st}^\lambda,$$

where

$$\alpha(\lambda') = \frac{1}{2} \sum_{s=0}^{d-1} \sum_{i \geq 1} (\lambda_i^{(s)\prime} - 1) \lambda_i^{(s)\prime}$$

and

$$X_{st}^\lambda = \prod_{(i,j) \in [\lambda^{(t)}]} (q^{j-i} Q_t - Q_s) \cdot \prod_{(i,j) \in [\lambda^{(s)}]} \left((q^{j-i} Q_s - q^{\lambda_1^{(t)}} Q_t) \prod_{k=1}^{\lambda_1^{(t)}} \frac{q^{j-i} Q_s - q^{k-1-\lambda_k^{(t)\prime}} Q_t}{q^{j-i} Q_s - q^{k-\lambda_k^{(t)\prime}} Q_t} \right).$$

The formula by Geck, Iancu and Malle is more symmetric, and describes the Schur elements in terms of *beta numbers*. If $\lambda = (\lambda^{(0)}, \lambda^{(1)}, \ldots, \lambda^{(d-1)})$ is a d-partition of r, then the *length of* λ is $\ell(\lambda) = \max\{\ell(\lambda^{(s)}) \mid 0 \leq s \leq d-1\}$. Fix an integer L such that $L \geq \ell(\lambda)$. The *L-beta*

numbers for $\lambda^{(s)}$ are the integers $\beta_i^{(s)} = \lambda_i^{(s)} + L - i$ for $i = 1, \ldots, L$. Set $B^{(s)} = \{\beta_1^{(s)}, \ldots, \beta_L^{(s)}\}$ for $s = 0, \ldots, d-1$. The matrix $B = (B^{(s)})_{0 \le s \le d-1}$ is called the *L-symbol* of λ.

Theorem 4.2. *Let $\lambda = (\lambda^{(0)}, \ldots, \lambda^{(d-1)})$ be a d-partition of r with L-symbol $B = (B^{(s)})_{0 \le s \le d-1}$, where $L \ge \ell(\lambda)$. Let $a_L := r(d-1) + \binom{d}{2}\binom{L}{2}$ and $b_L := dL(L-1)(2dL - d - 3)/12$. Then*

$$s_\lambda(\mathbf{q}) = (-1)^{a_L} x^{b_L} (q-1)^{-r} (Q_0 Q_1 \ldots Q_{d-1})^{-r} \nu_\lambda / \delta_\lambda,$$

where

$$\nu_\lambda = \prod_{0 \le s < t \le d-1} (Q_s - Q_t)^L \prod_{0 \le s,\, t \le d-1} \prod_{b_s \in B^{(s)}} \prod_{1 \le k \le b_s} (q^k Q_s - Q_t)$$

and

$$\delta_\lambda = \prod_{0 \le s < t \le d-1} \prod_{(b_s, b_t) \in B^{(s)} \times B^{(t)}} (q^{b_s} Q_s - q^{b_t} Q_t) \prod_{0 \le s \le d-1} \prod_{1 \le i < j \le L} (q^{b_i^{(s)}} Q_s - q^{b_j^{(s)}} Q_s).$$

As the reader may see, in both formulas above, the factors of $s_\lambda(\mathbf{q})$ are not obvious. Hence, it is not obvious for which values of $\mathbf{q}$ the Schur element $s_\lambda(\mathbf{q})$ becomes zero.

5. A cancellation-free formula

In this section, we will give a cancellation-free formula for the Schur elements of $\mathcal{H}_{d,r}$. This formula is also symmetric.

Let $\lambda = (\lambda^{(0)}, \lambda^{(1)}, \ldots, \lambda^{(d-1)})$ be a d-partition of r. The multiset $(\lambda_i^{(s)})_{0 \le s \le d-1,\, i \ge 1}$ is a composition of r (*i.e.* a multiset of non-negative integers whose sum is equal to r). By reordering the elements of this composition, we obtain a partition of r. We denote this partition by $\bar{\lambda}$. (e.g., if $\lambda = ((4,1), \emptyset, (2,1))$, then $\bar{\lambda} = (4,2,1,1)$).

Theorem 5.1. *Let $\lambda = (\lambda^{(0)}, \lambda^{(1)}, \ldots, \lambda^{(d-1)})$ be a d-partition of r. Then*

$$s_\lambda(\mathbf{q}) = (-1)^{r(d-1)} q^{-n(\bar{\lambda})} (q-1)^{-r} \prod_{s=0}^{d-1} \prod_{(i,j) \in [\lambda^{(s)}]} \prod_{t=0}^{d-1} (q^{h_{i,j}^{\lambda^{(t)}}} Q_s Q_t^{-1} - 1). \tag{1}$$

Since the total number of nodes in λ is equal to r, the above formula can be rewritten as follows:

$$s_\lambda(\mathbf{q}) = (-1)^{r(d-1)} q^{-n(\bar{\lambda})} \prod_{0 \le s \le d-1} \prod_{(i,j) \in [\lambda^{(s)}]} \left([h_{i,j}^{\lambda^{(s)}}]_q \prod_{0 \le t \le d-1,\, t \ne s} (q^{h_{i,j}^{\lambda^{(t)}}} Q_s Q_t^{-1} - 1) \right). \tag{2}$$

We will now proceed to the proof of the above result. Following Theorem 4.1, we have that

$$s_\lambda(\mathbf{q}) = (-1)^{r(d-1)} (Q_0 Q_1 \cdots Q_{d-1})^{-r} q^{-\alpha(\lambda')} \prod_{s=0}^{d-1} \prod_{(i,j) \in [\lambda^{(s)}]} Q_s [h_{i,j}^{\lambda^{(s)}}]_q \cdot \prod_{0 \le s < t \le d-1} X_{st}^{\lambda},$$

where

$$\alpha(\lambda') = \frac{1}{2} \sum_{s=0}^{d-1} \sum_{i \ge 1} (\lambda_i^{(s)'} - 1) \lambda_i^{(s)'}$$

and

$$X^{\lambda}_{st} = \prod_{(i,j)\in[\lambda^{(t)}]} (q^{j-i}Q_t - Q_s) \cdot \prod_{(i,j)\in[\lambda^{(s)}]} \left((q^{j-i}Q_s - q^{\lambda_1^{(t)}}Q_t) \prod_{k=1}^{\lambda_1^{(t)}} \frac{q^{j-i}Q_s - q^{k-1-\lambda_k^{(t)\prime}}Q_t}{q^{j-i}Q_s - q^{k-\lambda_k^{(t)\prime}}Q_t} \right).$$

The following lemma relates the terms $q^{-n(\bar{\lambda})}$ and $q^{-\alpha(\lambda')}$.

Lemma 5.2. *Let λ be a d-partition of r. We have that*

$$\alpha(\lambda') + \sum_{0\le s<t\le d-1} \sum_{i\ge 1} \lambda_i^{(s)\prime}\lambda_i^{(t)\prime} = n(\bar{\lambda}).$$

Proof. Following the definition of the conjugate partition, we have $\bar{\lambda}'_i = \sum_{s=0}^{d-1} \lambda_i^{(s)\prime}$, for all $i \ge 1$. Therefore,

$$\begin{aligned} n(\bar{\lambda}) &= \frac{1}{2}\sum_{i\ge1}(\bar{\lambda}'_i - 1)\bar{\lambda}'_i = \frac{1}{2}\sum_{i\ge 1}\left(\left(\sum_{s=0}^{d-1}\lambda_i^{(s)\prime} - 1\right)\cdot \sum_{s=0}^{d-1}\lambda_i^{(s)\prime}\right) \\ &= \frac{1}{2}\sum_{i\ge1}\left(\sum_{0\le s<t\le d-1} 2\cdot \lambda_i^{(s)\prime}\lambda_i^{(t)\prime} + \sum_{s=0}^{d-1}\lambda_i^{(s)\prime 2} - \sum_{s=0}^{d-1}\lambda_i^{(s)\prime}\right) \\ &= \sum_{0\le s<t\le d-1}\sum_{i\ge1}\lambda_i^{(s)\prime}\lambda_i^{(t)\prime} + \frac{1}{2}\sum_{s=0}^{d-1}\sum_{i\ge1}(\lambda_i^{(s)\prime}-1)\lambda_i^{(s)\prime} = \sum_{0\le s<t\le d-1}\sum_{i\ge1}\lambda_i^{(s)\prime}\lambda_i^{(t)\prime} + \alpha(\lambda') \end{aligned}$$

□

Hence, to prove Equality (2), it is enough to show that, for all $0 \le s < t \le d-1$,

$$X^{\lambda}_{st} = q^{-\sum_{i\ge1}\lambda_i^{(s)\prime}\lambda_i^{(t)\prime}} Q_s^{|\lambda^{(t)}|} Q_t^{|\lambda^{(s)}|} \prod_{(i,j)\in[\lambda^{(s)}]} (q^{h^{\lambda^{(t)}}_{i,j}} Q_s Q_t^{-1} - 1) \cdot \prod_{(i,j)\in[\lambda^{(t)}]} (q^{h^{\lambda^{(s)}}_{i,j}} Q_t Q_s^{-1} - 1). \tag{3}$$

We will proceed by induction on the number of nodes of $\lambda^{(s)}$. We do not need to do the same for $\lambda^{(t)}$, because the symmetric formula for the Schur elements given by Theorem 4.2 implies the following: if μ is the multipartition obtained from λ by exchanging $\lambda^{(s)}$ and $\lambda^{(t)}$, then

$$X^{\lambda}_{st}(Q_s, Q_t) = X^{\mu}_{st}(Q_t, Q_s).$$

If $\lambda^{(s)} = \emptyset$, then

$$\begin{aligned} X^{\lambda}_{st} &= \prod_{(i,j)\in[\lambda^{(t)}]}(q^{j-i}Q_t - Q_s) = Q_s^{|\lambda^{(t)}|}\prod_{(i,j)\in[\lambda^{(t)}]}(q^{j-i}Q_tQ_s^{-1} - 1) = Q_s^{|\lambda^{(t)}|}\prod_{i=1}^{\lambda_1^{(t)\prime}}\prod_{j=1}^{\lambda_i^{(t)}}(q^{j-i}Q_tQ_s^{-1} - 1) \\ &= Q_s^{|\lambda^{(t)}|}\prod_{i=1}^{\lambda_1^{(t)\prime}}\prod_{j=1}^{\lambda_i^{(t)}}(q^{\lambda_i^{(t)}-j+1-i}Q_tQ_s^{-1} - 1) = Q_s^{|\lambda^{(t)}|}\prod_{(i,j)\in[\lambda^{(t)}]}(q^{h^{\lambda^{(s)}}_{i,j}}Q_tQ_s^{-1} - 1), \end{aligned}$$

as required.

Now, assume that our assertion holds when $\#[\lambda^{(s)}] \in \{0, 1, 2, \dots, N-1\}$. We want to show that it also holds when $\#[\lambda^{(s)}] = N \geq 1$. If $\lambda^{(s)} \neq \emptyset$, then there exists i such that $(i, \lambda_i^{(s)})$ is a removable node of $\lambda^{(s)}$. Let ν be the multipartition defined by

$$\nu_i^{(s)} := \lambda_i^{(s)} - 1, \ \ \nu_j^{(s)} := \lambda_j^{(s)} \text{ for all } j \neq i, \ \ \nu^{(t)} := \lambda^{(t)} \text{ for all } t \neq s.$$

Then $[\lambda^{(s)}] = [\nu^{(s)}] \cup \{(i, \lambda_i^{(s)})\}$. Since Equality (3) holds for X_{st}^{ν} and

$$X_{st}^{\lambda} = X_{st}^{\nu} \cdot \left((q^{\lambda_i^{(s)} - i} Q_s - q^{\lambda_1^{(t)}} Q_t) \prod_{k=1}^{\lambda_1^{(t)}} \frac{q^{\lambda_i^{(s)} - i} Q_s - q^{k-1-\lambda_k^{(t)\prime}} Q_t}{q^{\lambda_i^{(s)} - i} Q_s - q^{k-\lambda_k^{(t)\prime}} Q_t} \right),$$

it is enough to show that (to simplify notation, from now on set $\lambda := \lambda^{(s)}$ and $\mu := \lambda^{(t)}$):

$$(q^{\lambda_i - i} Q_s - q^{\mu_1} Q_t) \prod_{k=1}^{\mu_1} \frac{q^{\lambda_i - i} Q_s - q^{k-1-\mu_k'} Q_t}{q^{\lambda_i - i} Q_s - q^{k-\mu_k'} Q_t} = q^{-\mu_{\lambda_i}'} Q_t (q^{\lambda_i - i + \mu_{\lambda_i}' - \lambda_i + 1} Q_s Q_t^{-1} - 1) \cdot A \cdot B, \qquad (4)$$

where

$$A := \prod_{k=1}^{\lambda_i - 1} \frac{q^{\lambda_i - i + \mu_k' - k + 1} Q_s Q_t^{-1} - 1}{q^{\lambda_i - i + \mu_k' - k} Q_s Q_t^{-1} - 1}$$

and

$$B := \prod_{k=1}^{\mu_{\lambda_i}'} \frac{q^{\mu_k - k + \lambda_{\lambda_i}' - \lambda_i + 1} Q_t Q_s^{-1} - 1}{q^{\mu_k - k + \lambda_{\lambda_i}' - \lambda_i} Q_t Q_s^{-1} - 1}.$$

Note that, since (i, λ_i) is a removable node of λ, we have $\lambda_{\lambda_i}' = i$. We have that

$$A = q^{\lambda_i - 1} \prod_{k=1}^{\lambda_i - 1} \frac{q^{\lambda_i - i} Q_s - q^{k-1-\mu_k'} Q_t}{q^{\lambda_i - i} Q_s - q^{k-\mu_k'} Q_t}.$$

Moreover, by Lemma 2.1, for $y = q^{i - \lambda_i} Q_t Q_s^{-1}$, we obtain that

$$B = \frac{(q^{\mu_1 + i - \lambda_i} Q_t Q_s^{-1} - 1)}{(q^{-\mu_{\lambda_i}' + \lambda_i - 1 + i - \lambda_i} Q_t Q_s^{-1} - 1)} \cdot \left(\prod_{k=\lambda_i}^{\mu_1} \frac{q^{-\mu_k' + k - 1 + i - \lambda_i} Q_t Q_s^{-1} - 1}{q^{-\mu_k' + k + i - \lambda_i} Q_t Q_s^{-1} - 1} \right),$$

i.e.,

$$B = Q_t^{-1} q^{\mu_{\lambda_i}' - \lambda_i + 1} \frac{(q^{\lambda_i - i} Q_s - q^{\mu_1} Q_t)}{(q^{\mu_{\lambda_i}' - \lambda_i + 1 + \lambda_i - i} Q_s Q_t^{-1} - 1)} \cdot \left(\prod_{k=\lambda_i}^{\mu_1} \frac{q^{\lambda_i - i} Q_s - q^{k-1-\mu_k'} Q_t}{q^{\lambda_i - i} Q_s - q^{k-\mu_k'} Q_t} \right).$$

Hence, Equality (4) holds.

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